

Montrons

$$\bar{F}(t_i) = \prod_{j=1}^i (1 - d(t_j)) \quad \forall i$$

On a si $j \geq 2$, $d(t_j) = \frac{P_j}{\bar{F}(t_{j-1})}$ donc $1 - d(t_j) = 1 - \frac{P_j}{\bar{F}(t_{j-1})}$

$$\text{si } j=1 \quad d(t_1) = \frac{P_1}{\sum_{j: t_j \geq t_1} P_j} = P_1$$

$$\begin{aligned} \text{et } 1 - d(t_1) &= 1 - P_1 \\ &= \frac{P_2 + P_3 + \dots}{\bar{F}(t_1)} \end{aligned}$$

$$\begin{aligned} &= \frac{\bar{F}(t_{j-1}) - P_j}{\bar{F}(t_{j-1})} \\ &= \frac{P_j + P_{j+1} + \dots - P_j}{\bar{F}(t_{j-1})} \\ &= \frac{\bar{F}(t_j)}{\bar{F}(t_{j-1})} \end{aligned}$$

On a donc

$$\begin{aligned} \prod_{j=1}^i (1 - d(t_j)) &= \prod_{j=1}^i \frac{\bar{F}(t_j)}{\bar{F}(t_{j-1})} = \bar{F}(t_1) \frac{\bar{F}(t_2)}{\bar{F}(t_1)} \dots \frac{\bar{F}(t_i)}{\bar{F}(t_{i-1})} \\ &= \bar{F}(t_i) \end{aligned}$$