Numerical Simulation by ROM of a Network Model of Intracellular Calcium Concentration in Neurons

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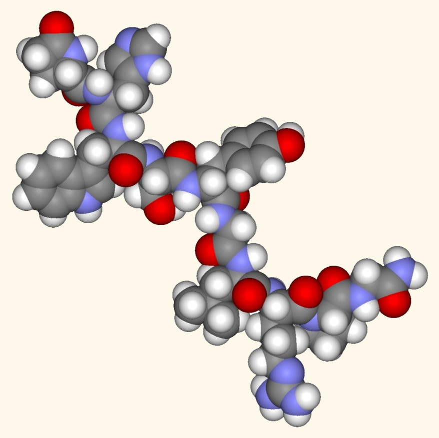
Introduction



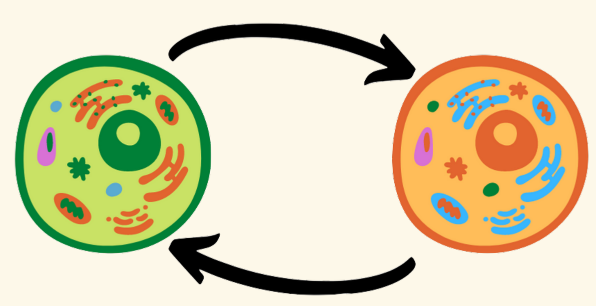
When reproducing the changes of Intracellular Calcium Concentration of Gonadotropin Releasing Hormone expressing neurons, the authors of [3] considered a system of 3D slow-fast oscillators, that are coupled through the fast variable in the slow dynamics of the recovery variable [3]. As a result, if we consider a higher number of cells in order to model network dynamics such us, for instance, the one presented in [1], we have to cope with a high-dimensional dynamical system that is computationally expensive to solve, so we apply a Reduced Order Model (ROM) approximation and present some results on the reduction. In a intermediate step, we study as well the behaviour of a pair of coupled heterogeneous cells. We get different behaviours than the ones obtained in the homogeneous case [2] and we present the new behaviours and how they are connected.

Model

N cells model: N coupled oscillators.

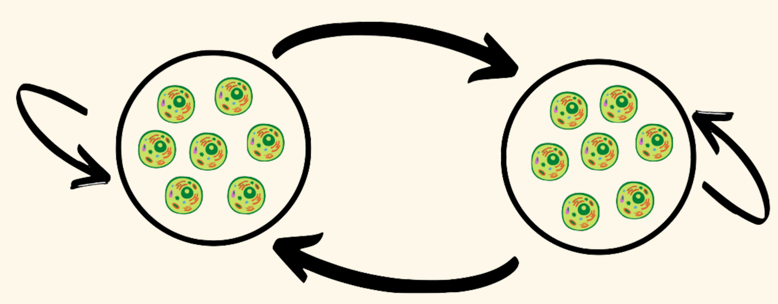
$$O_i \begin{cases} \dot{x}_i = \tau(-y_i + f(x_i) - \phi_f(z_i)), \\ \dot{y}_i = \tau\varepsilon k_i(x_i + a_1 y_i + a_2 + \dfrac{1}{N/2}\displaystyle\sum_{j=1}^{N} c_{ij}(x_i - x_j)), \\ \dot{z}_i = \tau\varepsilon\left(\phi_r(x_i) - \dfrac{z_i - z_b}{\tau_z}\right). \end{cases}$$

Two heterogeneous cells: $N = 2$ and $k_1 = 1$, $k_2 = k > 1$.



Two clusters: N cells divided into two clusters.

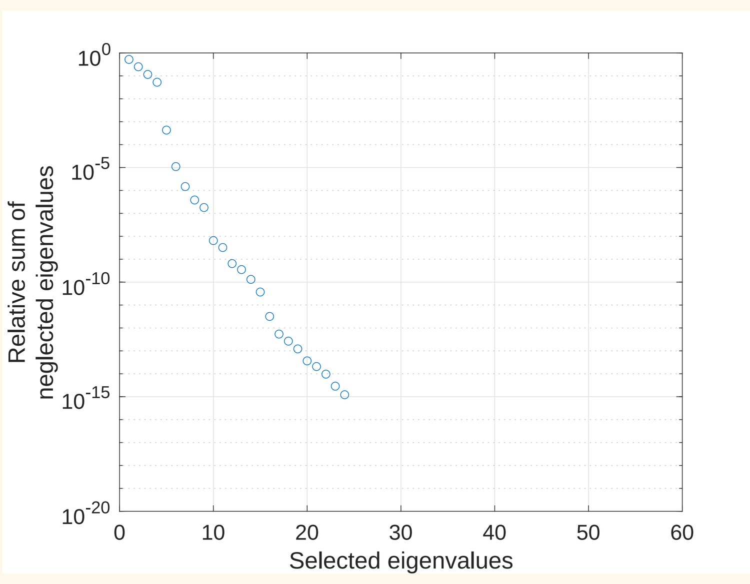
- Intracluster: In-phase, $c_{ij} = 1$.
- Intercluster: Antiphase, $c_{ij} \in [-0.45, -0.05]$.



The network has been constructed by coupling the one cell system in [3] with the structure of cells in [1].

ROM in the Two Clusters Case

Reduction in the number of equations for two cases: cells with same and different initial conditions.

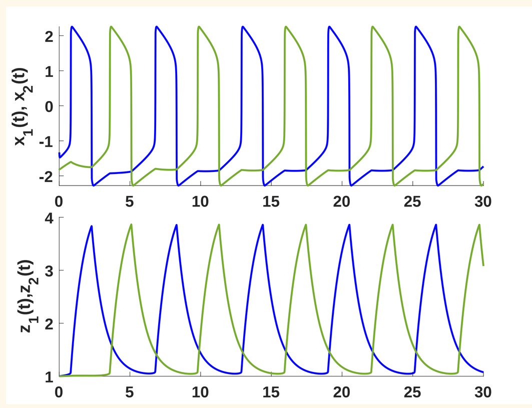


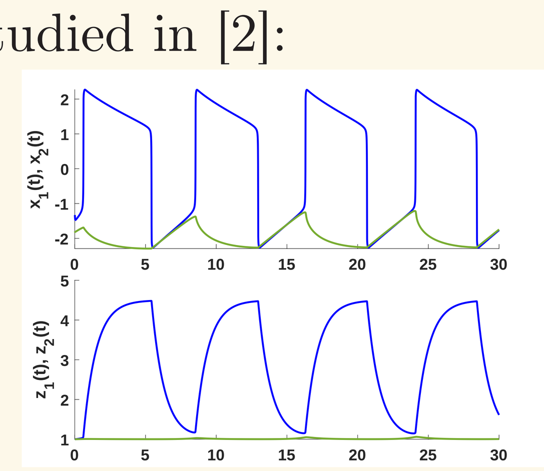
Initial Conditions	Model	ROM	%
Same	60	6	90%
Different	60	25	58.33%

Homogeneous Case

Behaviours in the homogeneous case $N = 2$ and $k = 1$ are studied in [2]:

- Antiphase synchronization for $c = -0.25$ (left).
- Relaxation loss for $c = -0.502$ (right).



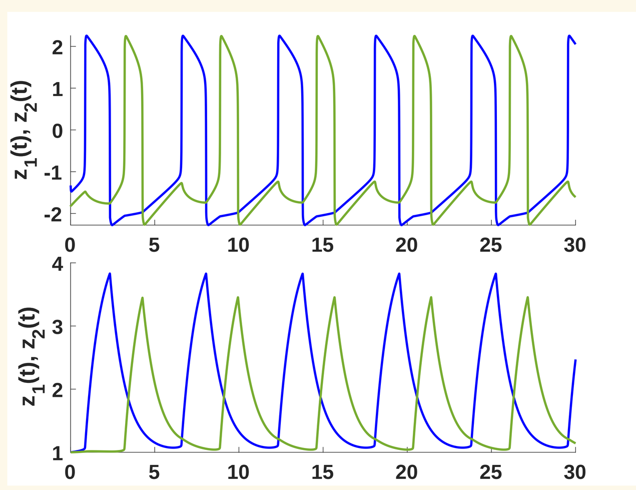


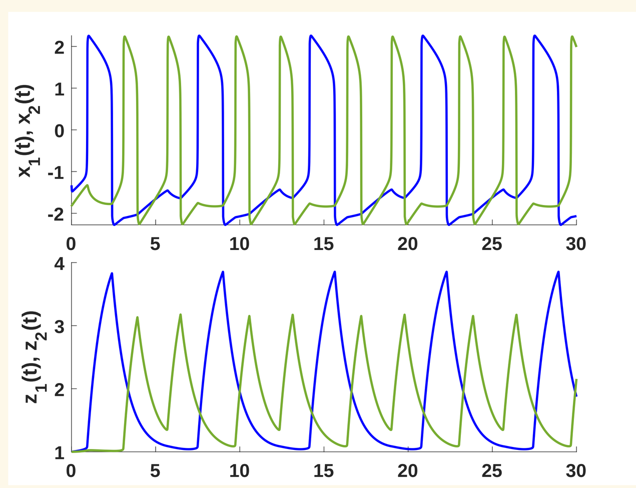
New Behaviour Patterns

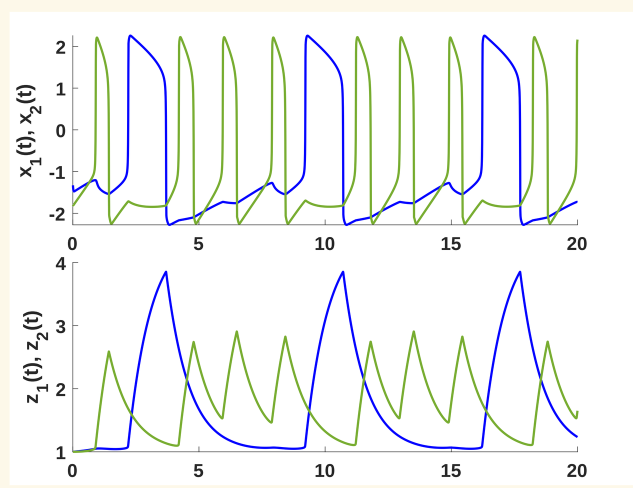
New behaviours in the heterogeneous case $N = 2$ and $k > 1$.

***s*/1**: An oscillator spikes s times while the other remain silent and then, the other oscillator spikes once, while the first remain silent.

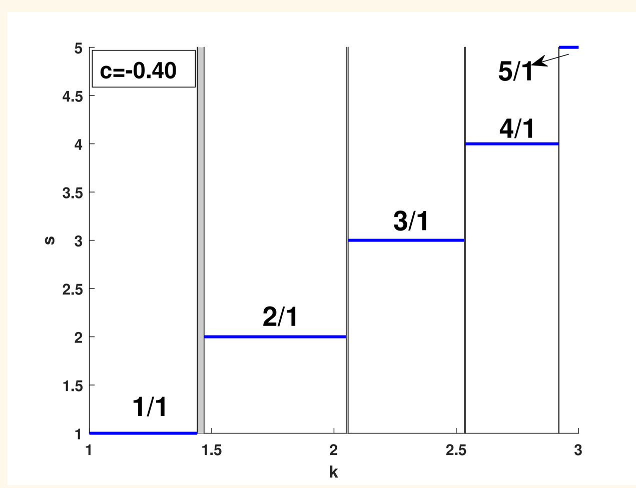
Selecting $c = -0.25$, we obtain: 1/1 for $k = 1.5$ (left), 2/1 for $k = 1.6$ (center), 3/1 for $k = 3$ (right).

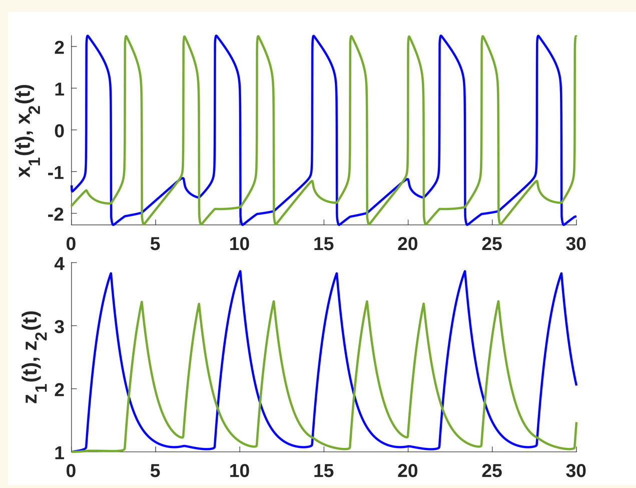


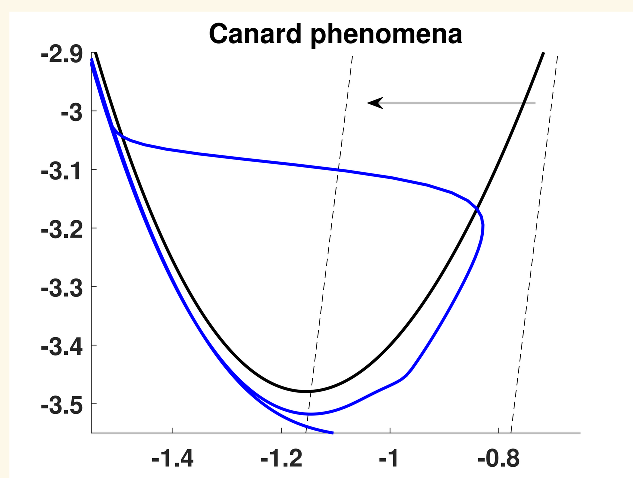




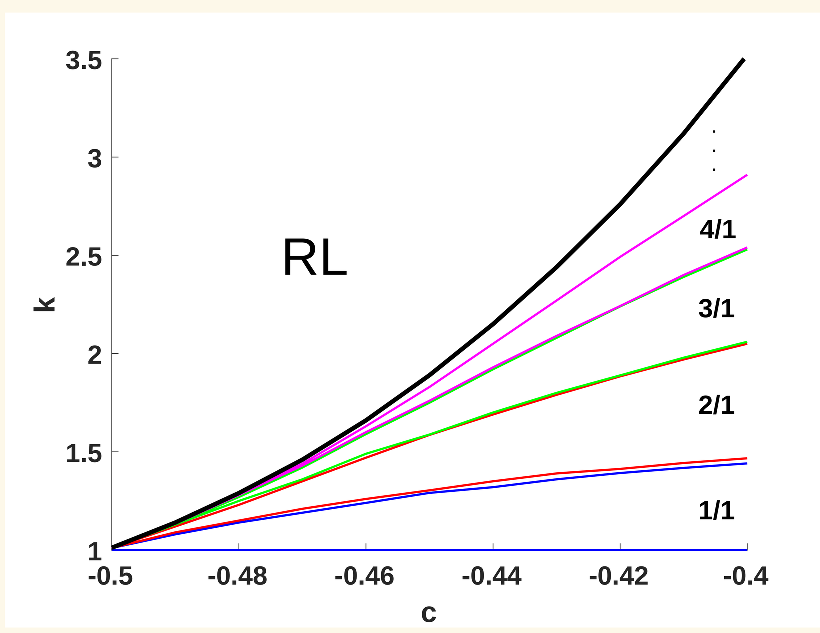
Devil's staircase: The system shows different stable behaviours $s/1$ separated by a transition part where the system can exhibit a mixture of stable behaviours or chaotic behaviour when the parameter k changes.

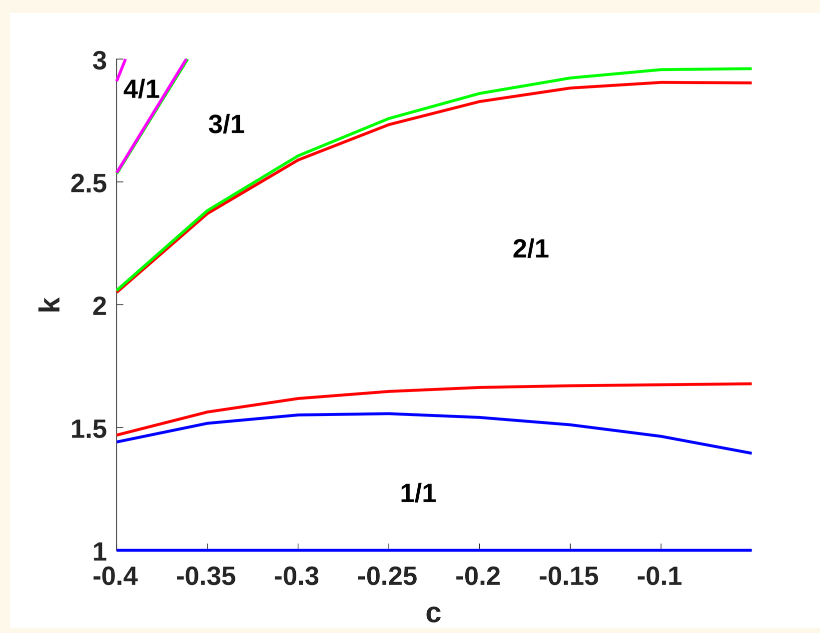






Transition to Relaxation Loss: Changes with the k value (left). Transitions length between main behaviours increases as c tends to zero.





References

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Conclusions

We have presented a model of a group of N cells and we have applied a ROM in the case where the cells are separated into two clusters.

In a numerical study of the heterogeneous system we have obtained new behaviours and that the system exhibit a devil-staircase-like pattern when changing the heterogeneity ratio between cells k_1/k_2. We have also obtained that the transition between the relaxation loss and anti-phase synchronization regime depends on the heterogeneity ratio.