The master Girsanov and Enlargement formulae

1) Girsanov formulae: $Q|_{\mathcal{F}_t} = M_t \cdot P|_{\mathcal{F}_t}$.

Under Q,
$$\boxed{N_t = \tilde{N}_t + \int_0^t \frac{d\langle M, N\rangle_s}{M_s}} \qquad (1)$$

gives the canonical decomposition of $(N_t, t \geq 0)$ a $(P, \mathcal{F}_t)$ continuous local martingale, as the sum of $(\tilde{N}_t, t \geq 0)$ its $(Q, \mathcal{F}_t)$ martingale part and $\left(\int_0^t \frac{d\langle M, N\rangle_s}{M_s}, t \geq 0\right)$ its bounded variation part.

Note also the companion formula
$$\boxed{N_t = \tilde{N}_t + \langle L, N\rangle_t} \qquad (2)$$
where $L_t = \int_0^t \frac{dM_s}{M_s}$, $t \geq 0$.

2) Progressive enlargement; making the end g of a $(\mathcal{F}_t)$ predictable set a stopping time in $\mathcal{F}_t^{(g)} = \mathcal{F}_t \vee \sigma(g \wedge t)$.

$$\boxed{N_t = \tilde{N}_t + \int_0^{t \wedge g} \frac{d\langle N, Z\rangle_s}{Z_s} + \int_g^t \frac{d\langle N, 1-Z\rangle_s}{1-Z_s}} \qquad (3)$$

where $Z_t = P(g > t \mid \mathcal{F}_t)$ is the supermartingale (in $(P, \mathcal{F}_t)$) associated with g.

4) Initial enlargement, making the variable L in the Brownian filtration $(\mathcal{F}_t)$ measurable at the origin of times in $\mathcal{F}_t^{\sigma(L)} = \mathcal{F}_t \vee \sigma(L)$.

Assume: $E[f(L) \mid \mathcal{F}_t] = \lambda_t(f) = E[f(L)] + \int_0^t \dot{\lambda}_s(f)\, dB_s$,

with: $\dot{\lambda}_s(dx) = \rho(s, x)\, \lambda_s(dx)$.

Then:
$$\boxed{N_t = \tilde{N}_t + \int_0^t \rho(L, s)\, d\langle N, B\rangle_s} \qquad (4).$$

when: $\int_0^t |\rho(L,s)|\, |d\langle N, B\rangle_s| < \infty$.